Biostatistics 615/815
Statistical Computing

Gonçalo Abecasis

Course Objective

- Introduce skills required for executing statistical computing projects
- Applications and examples mostly in C.
  - Can be easily translated into R, etc.
- But the focus is on an algorithmic way of thinking!

Part I: Key Algorithms

- Connectivity
- Sorting
- Searching
- Hashing
- Key data structures

Part II: Statistical Methods

- Random Numbers
- Markov-Chain Monte-Carlo
  - Metropolis-Hastings
  - Gibbs Sampling
- Function Optimization
  - Naïve algorithms
  - Newton's Methods
  - E-M algorithm
- Numerical Integration

Textbooks

- Algorithms in C
  - Sedgewick (1998)
  - 3rd edition printed in 1998
- Numerical Recipes in C
  - Press, Teukolsky, Vetterling, Flannery
  - 2nd edition printed in 2002

Assessment for 615

- Weekly Assignments
  - About 50% of the final mark
- 2 Exams
  - About 50% of the final mark

Assessment for 815

- Weekly Assignments
  - About 33% of the final mark
- 2 Exams
  - About 33% of the final mark
- Project, to be completed in pairs
  - About 33% of the final mark

Office Hours

- Please cross out times at which you are usually unavailable in the sheet that is going around …

- My office:
  - School of Public Health II, M4132
- My e-mail:
  - goncalo@umich.edu

Algorithms

- Methods for solving problems that are well suited to computer implementation
- Good algorithms make apparently impossible problems become simple

Algorithms are ideas ...

- Focus on approach to a problem
- Typically, the actual implementation could be take many different forms
  - Computer languages
  - Pen and paper

Example: DNA Sequence Matches

- When the Human Genome Project started, searching through the entire genome sequence seemed impractical…
- For example,
  - Searching for ~150 sequences of about 500bp each in ~3,000,000,000 bases of sequence would take ~3 hours with the original BLAST or FASTA3 algorithms

Example: DNA Sequence Matches

- Mullikin and colleagues (2001) described an improved algorithm, using hash tables, that could do this in < 2 seconds

- Reference:
  - Ning, Cox and Mullikin (2001) *Genome Research* **11:**1725-1729

Today's Lecture

- Introduce a "Connectivity problem" and some alternative solutions
- If you haven't done much programming before, don't worry too much about implementation details.
  - We'll fill these in later lectures.

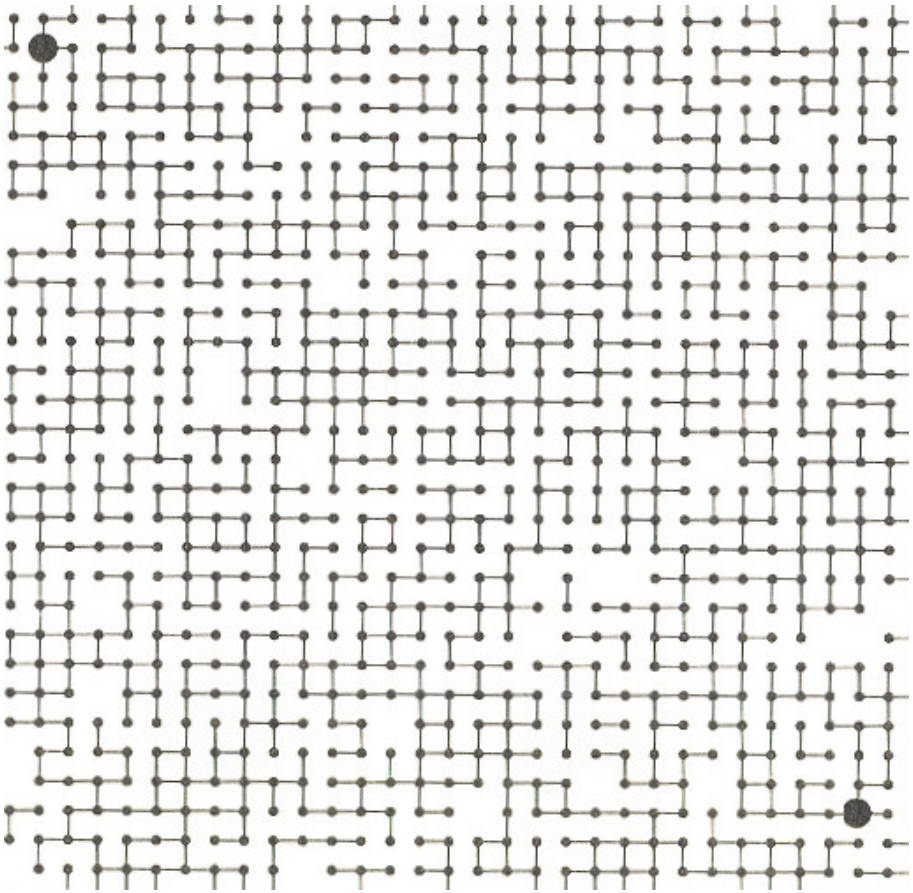
The Connectivity Problem

- N objects
  - Integer names 0 .. N – 1
- M connections between pairs of objects
  - Each connection identifies a pair *(p, q)*
- Possible questions:
  - Are all objects connected?
  - Are some connections redundant?
  - What are the groups of connected objects?

Possible applications

- Is a direct connection between two computers required in a network?
  - Or can we use some existing connections instead?
- Are two individuals part of the same extended family in a genetic study?
- Are two genes in the same regulatory network?

Are the two points connected?



Specific Question

- Can we identify redundant connections?
  - A redundant connection would link two points that are already connected
- For N objects there can be no more than N-1 non-redundant connections
  - Corresponds to all points being connected

A simple example ...

- Connections
  - 3-4
  - 4-9
  - 8-0
  - 2-3
  - 5-6
  - 2-9
  - 4-8
  - 0-2

A simple example ...

- Connections
  - 3-4 √
  - 4-9 √
  - 8-0 √
  - 2-3 √
  - 5-6 √
  - 2-9 Redundant: 2-3 ; 3-4 ; 4-9
  - 4-8 √
  - 0-2 Redundant: 0-8; 8-4; 4-3; 3-2

Specific Tasks

- As we proceed through list of connections, conduct two tasks:
  - Decide if each connection is new.
  - Incorporate information about new connections.

The Fundamental Operations

- The *Find* operation
  - Identify the set containing a particular item or items.
- The *Union* operation
  - Replace the sets containing two groups of objects by their union

The First Step

- Developing a solution that works
  - Easy to verify correctness
  - May not be most efficient
  - Should be simple
- Useful as check of “better” solutions…

Arrays of Integers

- Simple data structure
  - Analogous to a vector
- The notation `a[i]` refers to the `i`th integer in the array
  - When programming, we typically pre-specify the total number of entries in a array

Quick Find Algorithm

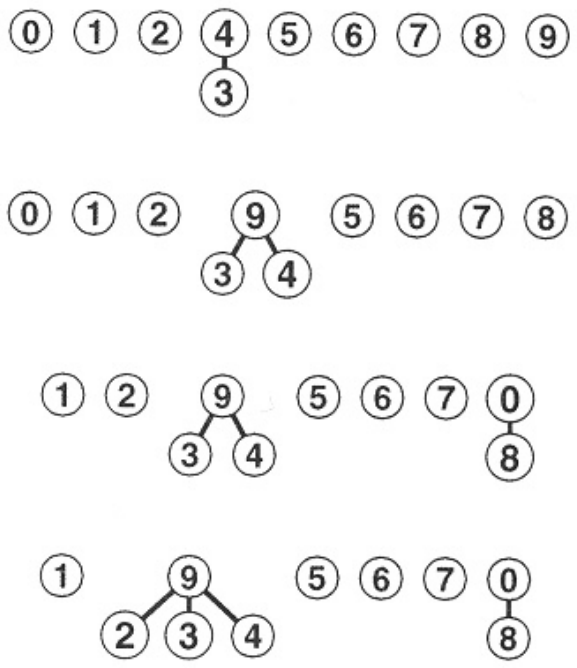
- Data
  - Array of N integers
  - Objects p and q connected iif `a[p] == a[q]`
- Setup
  - Initialize `a[i] = i`, for $0 \le i < N$
- For each pair
  - If `a[p] == a[q]` objects are connected (FIND)
  - Move all entries in set `a[p]` to set `a[q]` (UNION)

A Simple C Implementation

```
#define N 1000

int main()
   {
   int i, p, q, set, a[N];              // Variable declarations
   int unique_connections = 0;

   for (i = 0; i < N; i++)              // Data Initialization
      a[i] = i;

   while (scanf(" %d %d", &p, &q) == 2) // Loop through connections
      {
      if (a[p] == a[q]) continue;       // FIND operation

      set = a[p];                       // UNION operation
      for (i = 0; i < N; i++)
         if (a[i] == set)
            a[i] = a[q];

      printf("%d %d is a new connection\n", p, q);
      unique_connections++;
      }
   return 0;
   }
```

Pictorial Representation

- Array as connections are added:
  - 3-4
  - 4-9
  - 8-0
  - 2-3
  - 2-9 * Redundant *



How efficient is Quick Find?

- If there N objects and M connections*, the Quick Find algorithm requires on the order of MN operations
- Not feasible for very large numbers of objects…

* In this case only non-redundant connections actually count

Quick-Union Algorithm I

- Complementary to Quick Find
- More complex data organization
  - Each object points to “parent” object in the same set

Quick-Union Algorithm II

- For each pair
  - Follow pointers until we reach object that points to itself
  - If `a[p]` and `a[q]` eventually lead to the same object, we are in the same set (FIND)
  - Otherwise, link the object to which `a[p]` leads to the object which `a[q]` leads (UNION)

C Implementation

```
// Loop through connections
while (scanf(" %d %d", &p, &q) == 2)
   {
   // Check that input is within bounds
   if (p < 0 || p >= N || q < 0 || q >= N) continue;

   // FIND operation
   for (i = a[p]; a[i] != i; i = a[i] )  ;
   for (j = a[q]; a[j] != j; j = a[j] )  ;
   if (i == j) continue;

   // UNION operation
   a[i] = j;

   printf("%d %d is a new connection\n", p, q);
   unique_connections++;
   }
```

Pictorial Representation

- Array as connections are added:
  - 3-4
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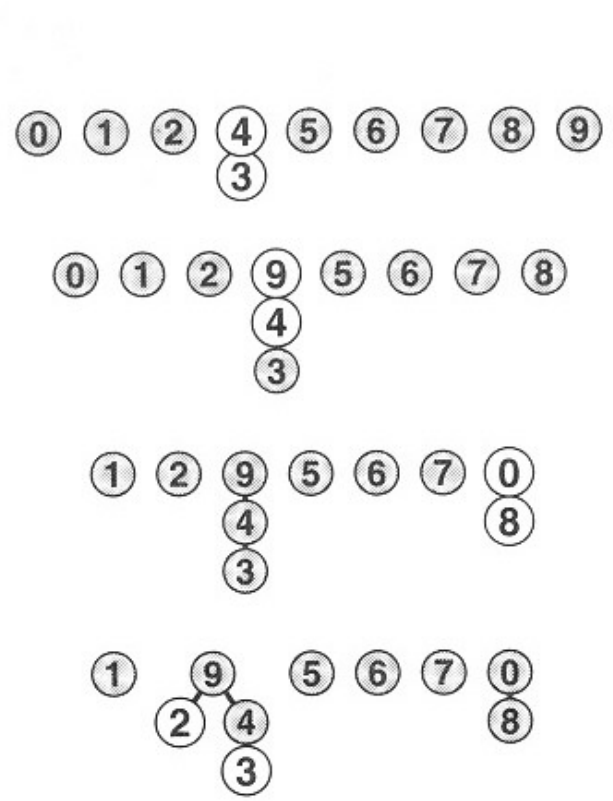

How efficient is Quick Union?

- Quick Union is typically faster than Quick Find.
- However, the data can conspire to make things difficult:
  - If objects are paired 1-2; 2-3; 3-4; 4-5; … we'll build long chains which slow down FIND operations
- In the worst case, we can still need about MN operations

Weighted Quick Union

- A smarter version of Quick Union, that avoids long chains
- Keep track of the number of elements in each set (using a separate array)
- Link smaller set to larger set
  - Union increases length of chains in smaller set by 1

C Implementation

```
// Initialize weights
for (i = 0; i < N; i++)
   weight[i] = 1;

// Loop through connections
while (scanf(" %d %d", &p, &q) == 2)
   {
   // Check that input is within bounds
   if (p < 0 || p >= N || q < 0 || q >= N) continue;

   // FIND operation
   for (i = a[p]; a[i] != i; i = a[i] )  ;
   for (j = a[q]; a[j] != j; j = a[j] )  ;
   if (i == j) continue;

   // UNION operation
   if (weight[i] < weight[j])
      { a[i] = j; weight[j] += weight[i]; }
   else
      { a[j] = i; weight[i] += weight[j]; }

   printf("%d %d is a new connection\n", p, q);
   unique_connections++;
   }
```

Pictorial Representation

- Array as connections are added:
  - 3-4
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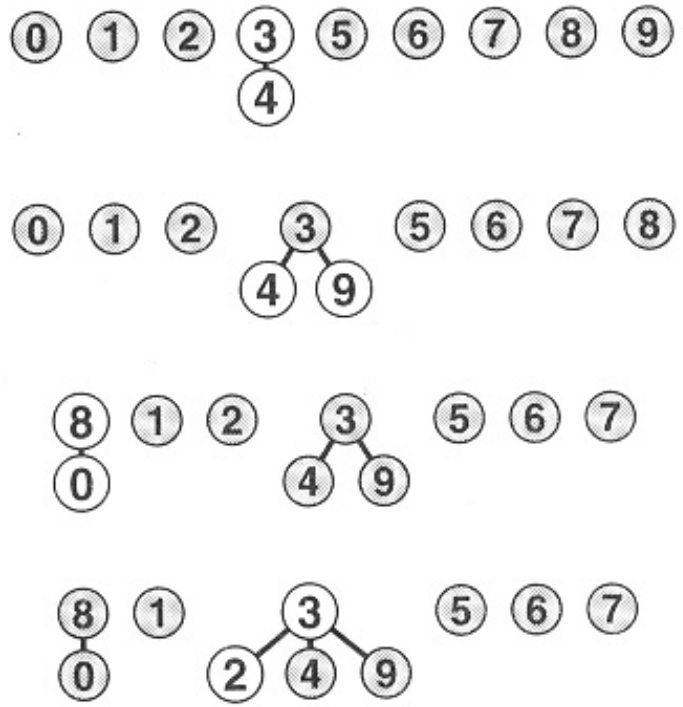
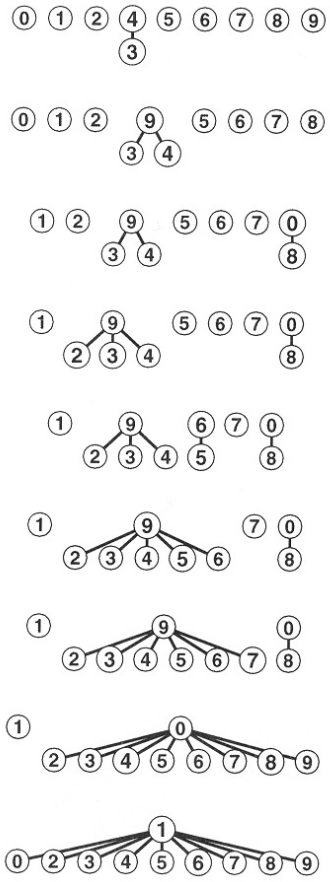


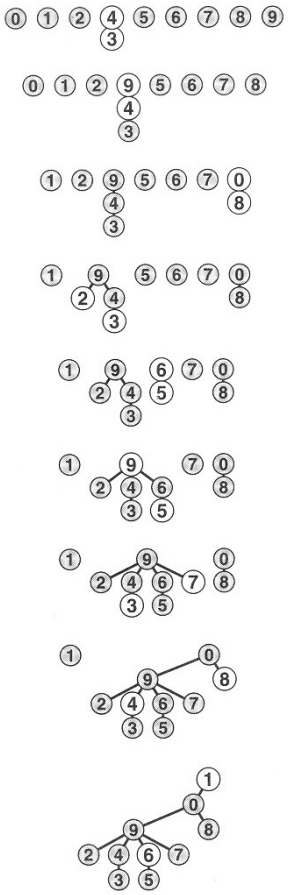
Efficiency of Weighted Quick Union

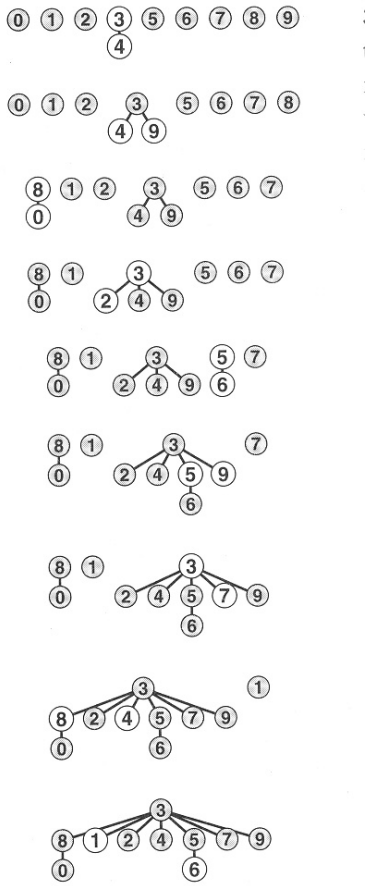
- Guarantees that pointer chains are no more than $\log_2 N$ elements long
- Overall, requires about $M \log_2 N$ operations
- Suitable for very large data sets with millions of objects and connections

Pictorial Comparison

Quick Union Quick Find Weighted







Empirical Timings in Seconds

Nodes (Connections)	Quick Find	Quick Union	Weighted Quick Union
50,000 (50,000)	6	1	<1
100,000 (100,000)	12	4	<1
200,000 (200,000)	25	15	<1

Summary

- Considered 3 alternative solutions to the “connectivity problem”
  - Are any connections in a set redundant?
  - Are all objects in a set connected?
- Compared some of the computational cost for the different methods

Reading Material

- Read Chapter 1 of Sedgewick

- www.sph.umich.edu/csg/abecasis/class/