Last Lecture ...

- **Recursive Functions**
  - Natural expression for many algorithms

- **Dynamic Programming**
  - Automatic strategy for generating efficient versions of recursive algorithms
Today ...

- Properties of Sorting Algorithms

- Elementary Sorting Algorithms
  - Selection Sort
  - Insertion Sort
  - Bubble Sort
Homework 1

- Limits of floating point

- Important concepts …
  - Precision is limited and relative
  - Errors can accumulate and lead to error
  - Mathematical soundness may not be enough
Floating Point Precision

- Smallest value that can be added to 1
  - $2^{-52}$ or $2.2 \times 10^{-16}$
  - $2^{-23}$ or $1.2 \times 10^{-7}$

- Smallest value that can be subtracted from 1
  - $2^{-53}$ or $1.1 \times 10^{-16}$
  - $2^{-24}$ or $6.0 \times 10^{-8}$

- Smallest value that is distinct from zero
  - $2^{-1074}$ or $4.9 \times 10^{-324}$
  - $2^{-149}$ or $1.4 \times 10^{-45}$
Calculating Powers of $\phi$

- Two possibilities

\[ \phi^n = \phi^{n-1} \phi \]

\[ \phi^n = \phi^{n-2} - \phi^{n-1} \]
Results Using Product Formula

Calculation Using Product

Exponent
Results Using Difference Formula
Relative Error on Log Scale

$\log(\frac{|product - difference|}{product})$

Exponent

Logarithm of Relative Error
Applications of Sorting

- Facilitate searching
  - Building indices

- Identify quantiles of a distribution

- Identify unique values

- Browsing data
Elementary Methods

- Suitable for
  - Small datasets
  - Specialized applications

- Prelude to more complex methods
  - Illustrate ideas
  - Introduce terminology
  - Sometimes useful complement
... but beware!

- Elementary sorts are very inefficient
  - Typically, time requirements are $O(N^2)$

- Probably, most common inefficiency in scientific computing
  - Make programs “break” with large datasets
Aim

- Rearrange a set of keys
  - Using some predefined order
    - Integers
    - Doubles
    - Indices for records in a database

- Keys stored as array in memory
  - More complex sorts when we can only load part of the data
Basic Building Blocks

- An type for each element
  
  ```
  #define Item int
  ```

- Compare two elements

- Exchange two elements

- Compare and exchange two elements
Comparing Two Elements

- Define a function to compare two elements

  ```cpp
  bool isLess(Item a, Item b)
  { return a < b; }
  ```

- Alternative is to use macros, but I don’t recommend it

  ```cpp
  #define isLess(a,b)  ((a)<(b))
  ```
Exchanging Two Elements

- The best way is to use a C++ function

```cpp
void Exchange(Item & a, Item & b)
    { Item temp = a; a = b; b = temp; }
```

- But using a macro is still an alternative

```cpp
#define Exchange(a,b) \
    { \
    Item tmp = (a); \
    (a) = (b); \
    (b) = tmp; \
    }
```
Comparing And Exchange

- Using C++ function

```cpp
Item CompExch(Item & a, Item & b)
{
    if (isLess(b, a))
        Exchange(a, b);
}
```

- Using a macro

```cpp
#define CompExch(a,b) \
    if (isLess((b),(a))) Exchange((a),(b));
```
A Simple Sort

- Gradually sort the array by:
  - Sorting the first 2 elements
  - Sorting the first 3 elements
  - ...
  - Sort all N elements
A Simple Sort Routine

```c
void sort(Item a[], int start, int stop)
{
    int i, j;

    for (i = start + 1; i <= stop; i++)
        for (j = i; j > start; j--)
            CompExch(a[j-1], a[j]);
}
```
Properties of this Simple Sort

- **Non-adaptive**
  - Comparisons do not depend on data

- **Stable**
  - Preserves relative order for duplicates

- Requires $O(N^2)$ running time
Sorts We Will Examine Today

- Selection Sort
- Insertion Sort
- Bubble Sort
Recipe: Selection Sort

- Find the smallest element
  - Place it at beginning of array

- Find the next smallest element
  - Place it in the second slot

- ...

- ...
C Code: Selection Sort

```c
void sort(Item a[], int start, int stop)
{
    int i, j;

    for (i = start; i < stop; i++)
    {
        int min = i;
        for (j = i + 1; j < stop; j++)
            if (isLess(a[j], a[min])
                min = j;

        Exchange(a[i], a[min]);
    }
}
```
Selection Sort

**Notice:**

Each exchange moves element into final position.

Right portion of array looks random.
Properties of Selection Sort

- Running time does not depend on input
  - Random data
  - Sorted data
  - Reverse ordered data...
- Performs exactly N-1 exchanges
- Most time spent on comparisons
Recipe: Insertion Sort

- The “Simple Sort” we first considered

- Consider one element at a time
  - Place it among previously considered elements
  - Must move several elements to “make room”

- Can be improved, by “adapting to data”
Improvement I

- Decide when further comparisons are futile
- Stop comparisons when we reach a smaller element
- What speed improvement do you expect?
void sort(Item a[], int start, int stop) {
    int i, j;

    for (i = start + 1; i <= stop; i++)
        for (j = i; j > start; j--)
            if (isLess(a[j], a[j-1]))
                Exchange(a[j-1], a[j]);
            else
                break;
}

Improvement II

- Notice that inner loop continues until:
  - First element reached, or
  - Smaller element reached

- If smallest element is at the beginning…
  - Only one condition to check
Insertion Sort (II)

```c
void sort(Item a[], int start, int stop)
{
    int i, j;

    // This ensures that smallest element is at the beginning
    for (i = stop; i > start; i--)
        CompExch(a[i-1], a[i]);

    // Now, we don't need to check that j > start
    for (i = start + 2; i <= stop; i++)
    {
        int j = i;
        while (isLess(a[j], a[j-1]))
        {
            Exchange(a[j], a[j-1]);
            j--;
        }
    }
}
```
Improvement III

- The basic approach requires many exchanges involving each element

- Instead of carrying out many exchanges ...

- Find out position for the new element and shift elements to the right to make room
void sort(Item a[], int start, int stop) {
    int i, j;

    for (i = stop; i > start; i--)
        CompExch(a[i-1], a[i]);

    for (i = start + 2; i <= stop; i++)
        {
            int j = i;
            Item val = a[j];       // Store the value of new element
            while (isLess(val, a[j-1]))   // Proceed through larger elements
            {
                a[j] = a[j-1];       // Shifting things to the right …
                j--;
            }
            a[j] = val;           // Finally, insert new element in place
        }
}
Insertion Sort

**Notice:**

Elements in left portion of array can still change position.

Right remains untouched.
Properties of Insertion Sort

- Adaptive version running time depends on input
  - About 2x faster on random data
  - Improvement even greater on sorted data
  - Similar speed on reverse ordered data

- Stable sort
Recipe: Bubble Sort

- Pass through the array
  - Exchange elements that are out of order

- Repeat until done…

- Very “popular”
  - Very inefficient too!
C Code: Bubble Sort

```c
void sort(Item a[], int start, int stop)
{
    int i, j;

    for (i = start; i <= stop; i++)
        for (j = stop; j > i; j--)
            CompExch(a[j-1], a[j]);
}
```
Bubble Sort

Notice:

Each pass moves one element into position.

Right portion of array is partially sorted
Shaker Sort

Notice:

Things improve slightly if bubble sort alternates directions…
Notes on Bubble Sort

- Similar to non-adaptive Insertion Sort
  - Moves through unsorted portion of array

- Similar to Selection Sort
  - Does more exchanges per element

- Stop when no exchanges performed
  - Adaptive, but not as effective as Insertion Sort
Performance Characteristics

- Selection, Insertion, Bubble Sorts
- All quadratic
  - Running time differs by a constant
- Which sorts do you think are stable?
Selection Sort

- Exchanges
  - $N - 1$

- Comparisons
  - $N \times (N - 1) / 2$

- Requires about $N^2 / 2$ operations
- Ignoring updates to min variable
Adaptive Insertion Sort

- **Half - Exchanges**
  - About $N^2 / 4$ on average (random data)
  - $N \times (N - 1) / 2$ (worst case)

- **Comparisons**
  - About $N^2 / 4$ on average (random data)
  - $N \times (N - 1) / 2$ (worst case)

- Requires about $N^2 / 4$ operations
- Requires nearly linear time on sorted data
Bubble Sort

- Exchanges
  - $N \times (N - 1) / 2$

- Comparisons
  - $N \times (N - 1) / 2$

- Average case and worst case very similar, even for adaptive method
## Empirical Comparison

<table>
<thead>
<tr>
<th>N</th>
<th>Selection</th>
<th>Insertion</th>
<th>Insertion (adaptive)</th>
<th>Bubble</th>
<th>Shaker</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>5</td>
<td>7</td>
<td>4</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>2000</td>
<td>21</td>
<td>29</td>
<td>15</td>
<td>45</td>
<td>34</td>
</tr>
<tr>
<td>4000</td>
<td>85</td>
<td>119</td>
<td>62</td>
<td>182</td>
<td>138</td>
</tr>
</tbody>
</table>

(Running times in seconds)
Reading

- Sedgewick, Chapter 6