Shell Sort

Biostatistics 615/815

Lecture 6

Housekeeping Note: Homework Grading

- Weihua Guan is the GSI
- He requests that you e-mail him source code for your assignments to:

  wguan@umich.edu

- Thanks!

Last Lecture …

- Properties of Sorting Algorithms
  - Adaptive
  - Stable
- Elementary Sorting Algorithms
  - Selection Sort
  - Insertion Sort
  - Bubble Sort

"Stable" and "Unstable" Sorts

City	State	Season
Las Vegas	NV	All Year
Denver	CO	All Year
Traverse City	MI	Summer
Holland	MI	Summer
Boulder	CO	Winter

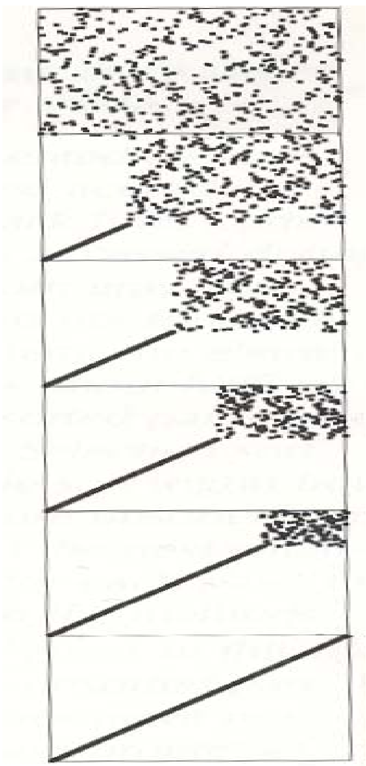
Stable Sort by State

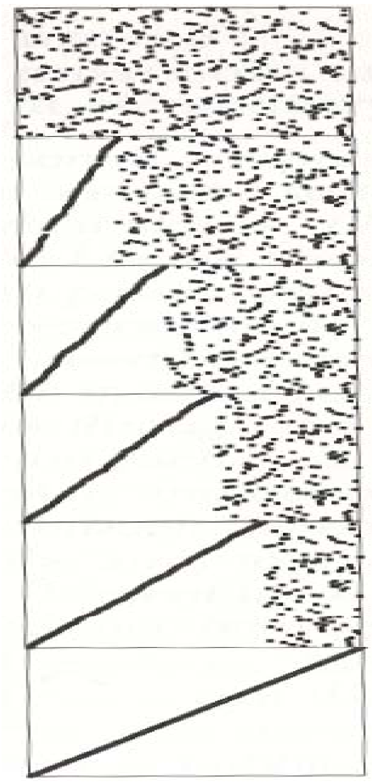
City	State	Season
Denver	CO	All Year
Boulder	CO	Winter
Traverse City	MI	Summer
Holland	MI	Summer
Las Vegas	NV	All Year

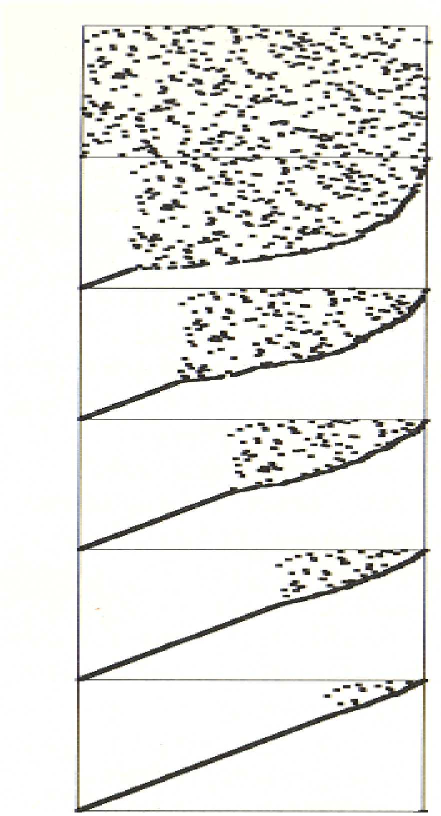
Unstable Sort by State

City	State	Season
Boulder	CO	Winter
Denver	CO	All Year
Holland	MI	Summer
Traverse City	MI	Summer
Las Vegas	NV	All Year

Selection Insertion Bubble







Recap

- Selection, Insertion, Bubble Sorts
- Can you think of:
  - One property that all of these share?
  - One useful advantage for Selection sort?
  - One useful advantage for Insertion sort?
- Situations where these sorts can be used?

Today …

- Shellsort
  - An algorithm that beats the $O(N^2)$ barrier
  - Suitable performance for general use

- Very popular
  - It is the basis of the default R sort() function

- Tunable algorithm
  - Can use different orderings for comparisons

Shellsort

- Donald L. Shell (1959)
  - *A High-Speed Sorting Procedure*
    Communications of the Association for Computing Machinery **2**:30-32
  - Systems Analyst working at GE
  - Back then, most computers read punch-cards
- Also called:
  - Diminishing increment sort
  - “Comb” sort
  - “Gap” sort

Intuition

- Insertion sort is effective:
  - For small datasets
  - For data that is nearly sorted
- Insertion sort is inefficient when:
  - Elements must move far in array

The Idea …

- Allow elements to move in large steps

- Bring elements close to final location
  - First, ensure array is nearly sorted …
  - … then, run insertion sort

- How?
  - Sort interleaved arrays first

Shellsort Recipe

- Decreasing sequence of step sizes *h*
  - Every sequence must end at 1
  - … , 8, 4, 2, 1

- For each h, sort sub-arrays that start at arbitrary element and include every h^{th} element
  - if h = 4
    - Sub-array with elements 1, 5, 9, 13 …
    - Sub-array with elements 2, 6, 10, 14 …
    - Sub-array with elements 3, 7, 11, 15 …
    - Sub-array with elements 4, 8, 12, 16 …

Shellsort Notes

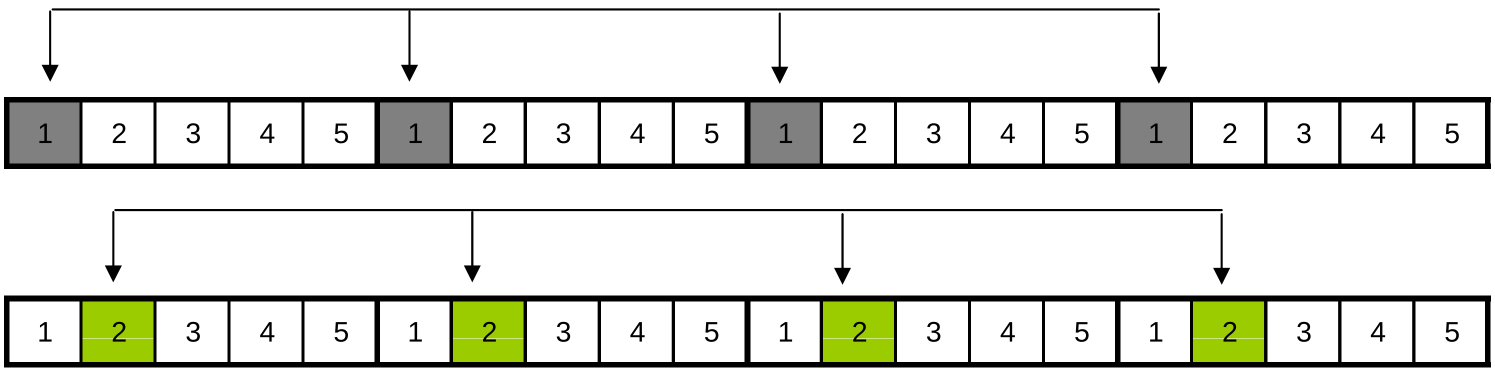
- Any decreasing sequence that ends at 1 will do…
  - The final pass ensures array is sorted

- Different sequences can dramatically increase (or decrease) performance

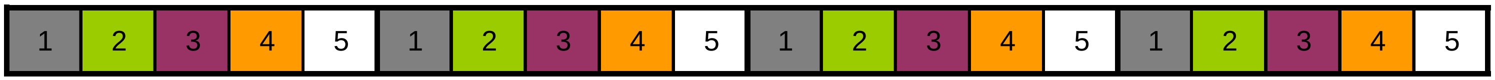
- Code is similar to insertion sort

Sub-arrays when Increment is 5

5-sorting an array



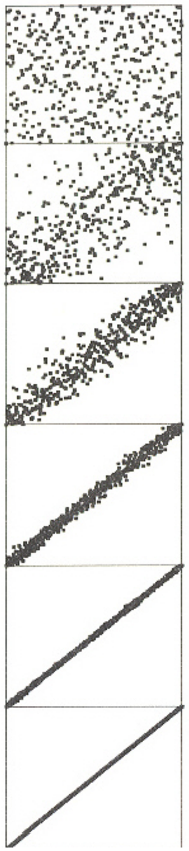
Elements in each subarray color coded



C Code: Shellsort

```
void sort(Item a[], int sequence[], int start, int stop)
   {
   int step, i;

   for (int step = 0; sequence[step] >= 1; step++)
      {
      int inc = sequence[step];

      for (i = start + inc; i <= stop; i++)
         {
         int j = i;
         Item val = a[i];

         while ((j >= start + inc) && val < a[j - inc])
            {
            a[j] = a[j - inc];
            j -= inc;
            }

         a[j] = val;
         }
      }
   }
```

Pictorial Representation



- Array gradually gains order
- Eventually, we approach the ideal case where insertion sort is O(N)

C Code: Using a Shell Sort

```
#include “stdlib.h”
#include “stdio.h”

#define Item int

void sort(Item a[], int sequence[], int start, int stop);

int main(int argc, char * argv[])
   {
   printf(“This program uses shell sort to sort a random array\n\n”);
   printf(“  Parameters: [array-size]\n\n”);

   int size = 100;
   if (argc > 1) size = atoi(argv[1]);

   int sequence[] = { 364, 121, 40, 13, 4,  1, 0};
   int * array = (int *) malloc(sizeof(int) * size);

   srand(123456);
   printf(“Generating %d random elements …\n”, size);
   for (int i = 0; i < size; i++)
       array[i] = rand();

   printf(“Sorting elements …\n”, size);
   sort(array, sequence, 0, size – 1);

   printf(“The sorted array is …\n”);
   for (int i = 0; i < size; i++)
      printf(“%d “, array[i]);
   printf(“\n”);
   free(array);
   }
```

Note on Example Code: Declaring Variables *“Late”*

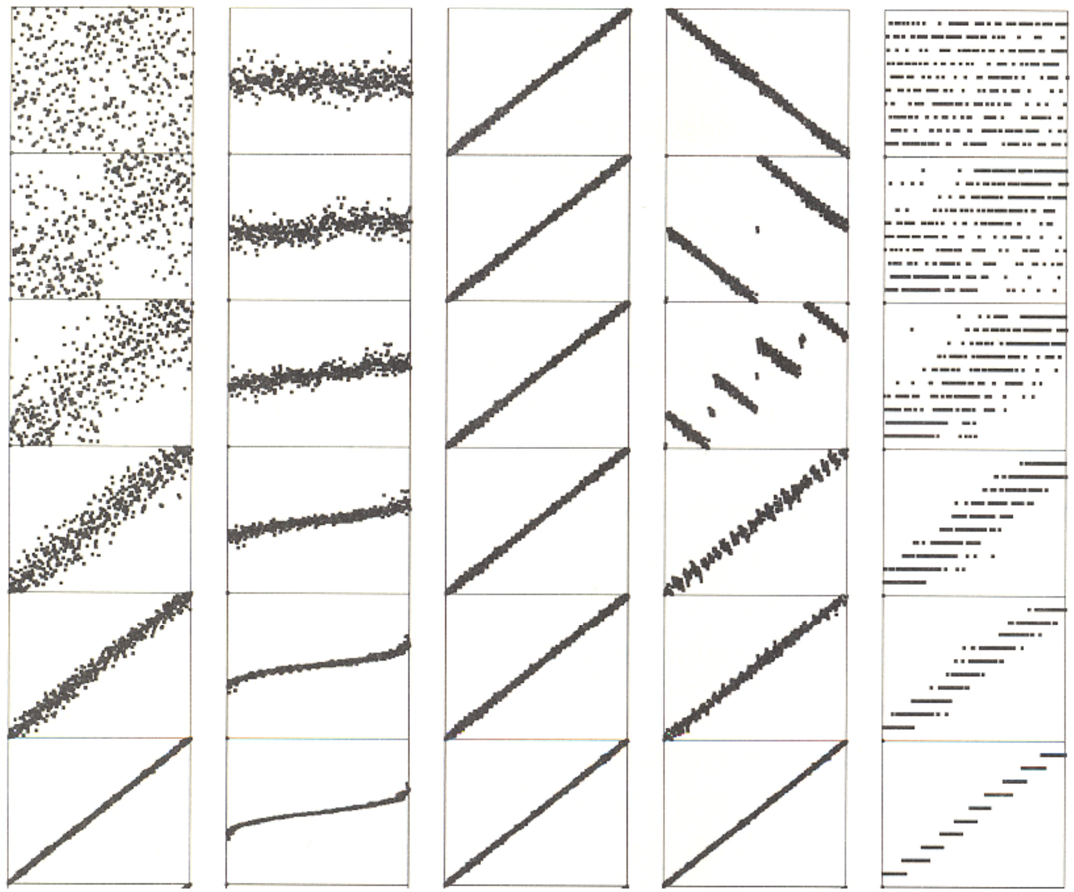
- Instead of declaring variables immediately after opening a {} block, wait until first use
  - Possibility introduced with C++
- Supported by most modern C compilers
  - In UNIX, use g++ instead of gcc to compile

Running Time (in seconds)

N	Pow2	Knuth	Merged	Seq1	Seq2
125000	1	0	0	0	0
250000	2	0	0	1	0
500000	6	1	1	0	1
1000000	14	2	2	1	2
2000000	42	5	2	4	3
4000000	118	10	6	7	8

Pow2 – 1, 2, 4, 8, 16 … (2^i)
Knuth – 1, 4, 13, 40, … (3 * *previous* + 1)
Seq1 – 1, 5, 41, 209, … ($4^i - 3 * 2^i + 1$)
Seq2 – 1, 19, 109, 505 … ($9 * 4^i - 9 * 2^i + 1$)
Merged – Alternate between Seq1 and Seq2

Not Sensitive to Input ...



Increment Sequences

- Good:
  - Consecutive numbers are relatively prime
  - Increments decrease roughly exponentially

- An example of a bad sequence:
  - 1, 2, 4, 8, 16, 32 …
  - What happens if the largest values are all in odd positions?

Shellsort Properties

- Not very well understood
- For good increment sequences, requires time proportional to
  - $N (\log N)^2$
  - $N^{1.25}$
- We will discuss them briefly …

Definition: *h*-Sorted Array

- An array where taking every h^{th} element (starting anywhere) yields a sorted array
- Corresponds to a set of several* sorted arrays interleaved together
  - * There could be *h* such arrays

Property I

- If we *h*-sort an array that is *k*-ordered…
- Result is an *h*- and *k*- ordered array

- *h*-sort preserves *k*-order!

- Seems tricky to prove, but considering a set of 4 elements as they are sorted in parallel makes things clear…

Property I

- Result of *h*-sorting an array that is *k*-ordered is an *h*- and *k*- ordered array

- Consider 4 elements, in k-ordered array:
  - a[i] <= a[i+k]
  - a[i+h] <= a[i+k+h]

- After h-sorting, a[i] contains minimum and a[i+k+h] contains maximum of all 4

Property II

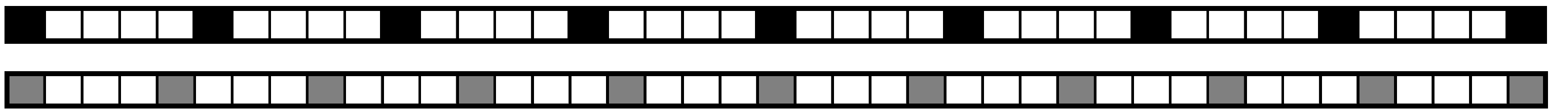
- **If *h* and *k* are relatively prime …**

- Items that are more than *(h-1)(k-1)* steps apart must be in order
  - Possible to step from one to the other using steps size *h* or *k*
  - That is, by stepping through elements known to be in order.

- Insertion sort requires no more *(h-1)(k-1)* comparisons per item to sort array that is *h*- and *k*-sorted
  - Or (h-1)(k-1)/g comparisons to carry a *g*-sort

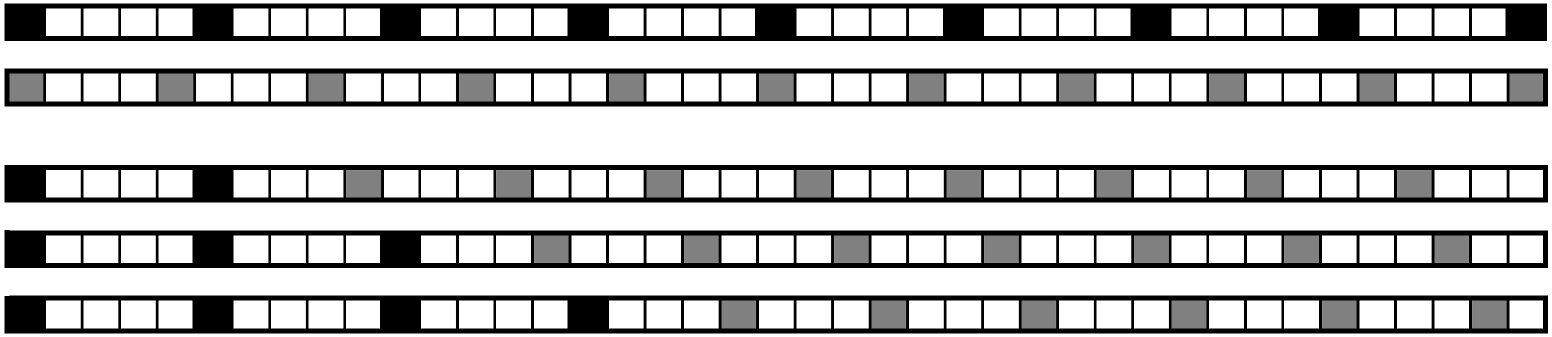
Property II

- Consider *h* and *k* sorted arrays
  - Say h = 4 and k = 5
- Elements that must be in order



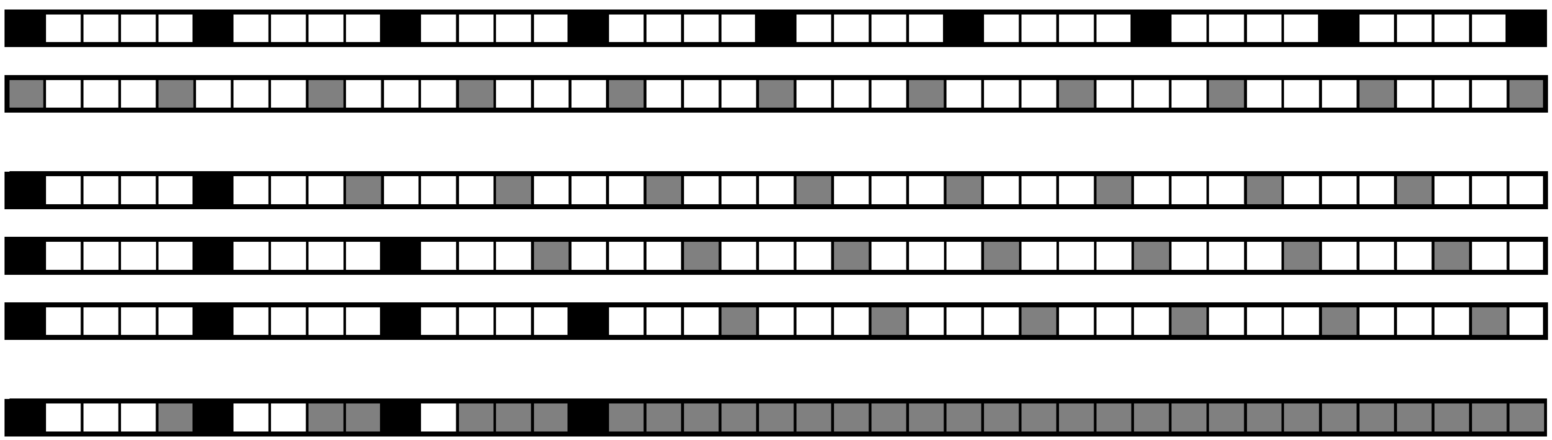
Property II

- Consider *h* and *k* sorted arrays
  - Say h = 4 and k = 5
- More elements that must be in order …



Property II

- Combining the previous series gives the desired property that elements (h-1)(k-1) elements away must be in order



An optimal series?

- Considering the two previous properties…
- A series where every sub-array is known to be 2- and 3- ordered could be sorted with a single round of comparisons
- Is it possible to construct series of increments that ensures this?
    - Before *h*-sorting, ensure 2*h* and 3*h* sort have been done …

Optimal Performance?

- Consider a triangle of increments:
  - Each element is:
    - double the number above to the right
    - three times the number above to the left
  - $< \log_2 N \log_3 N$ increments

					1					
				2		3				
			4		6		9			
		8		12		18		27		
	16		24		36		54		81	
32		48		72		108		162		243

Optimal Performance?

- Start from bottom to top, right to left
- After first row, every sub-array is 3-sorted and 2-sorted
  - No more than 1 exchange!
- In total, there are ~ $\log_2 N \log_3 N / 2$ increments
  - About $N (\log N)^2$ performance possible

Today's Summary: Shellsort

- Breaks the N^2 barrier
  - Does not compare all pairs of elements, ever!
- Average and worst-case performance similar
- Difficult to analyze precisely

Reading

- Sedgewick, Chapter 6