Quick Sort

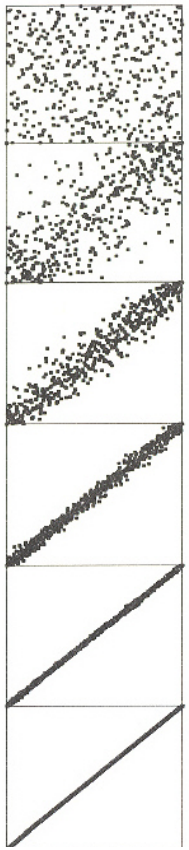
Biostatistics 615/815

Lecture 7

Last Lecture: Shell Sort

- Gradually bring order to array by:
  - Sorting sub-arrays including every k^{th} element
  - Using a series of step sizes *k*, ending with *k = 1*
- Each pass handles nearly sorted arrays where insertion sort is efficient
- Theoretically, N $(\log N)^2$ complexity is possible

Pictorial Representation



- Array gradually gains order
- Eventually, we approach the ideal case where insertion sort is O(N)

Today: Quick Sort

- Most widely used sorting algorithm
  - Possibly excluding those bubble sorts that should be banished!
- Extremely efficient
  - *O(N log N)*
- Divide-and-conquer algorithm

The Inventor of Quicksort

- Sir Charles A. R. Hoare
  - 1980 ACM Turing Award
- British computer scientist
  - Studied statistics as a graduate student
- Made major contributions to developing computer languages

C. A. R. Hoare Quote

“I conclude that there are two ways of constructing a software design:

One way is to make it so simple that there are obviously no deficiencies and the other way is to make it so complicated that there are no obvious deficiencies."

Caution!

- Quicksort is fragile!
  - Small mistakes can be hard to spot
  - Shellsort is more robust (but slower)
- The worst case running time is $O(N^2)$
  - Can be avoided in nearly all practical settings

Divide-And-Conquer

- Divide a problem into smaller sub-problems
- Find a partitioning element such that:
  - All elements to the right are greater
  - All elements to the left are smaller
  - Sort right and left sub-arrays independently

C Code: QuickSort

```
void quicksort(Item a[], int start, int stop)
  {
  int i;

  if (stop <= start) return;

  i = partition(a, start, stop);
  quicksort(a, start, i - 1);
  quicksort(a, i + 1, stop);
  }
```

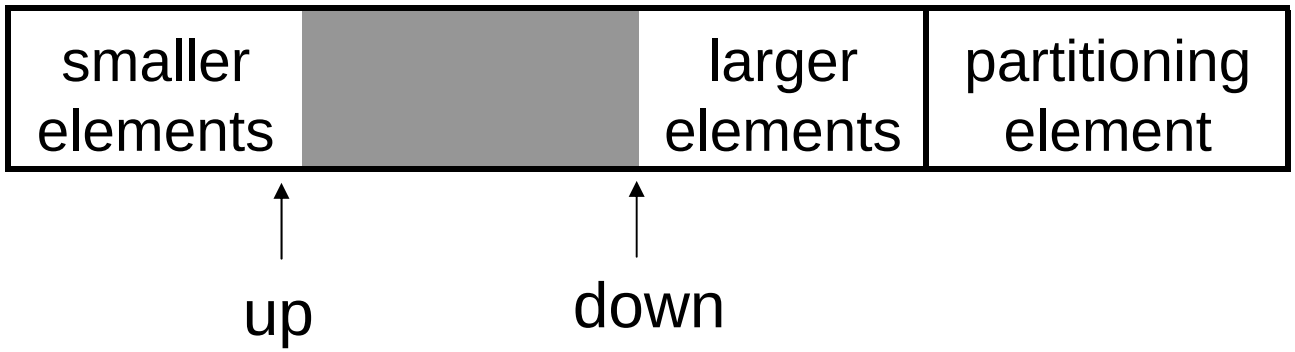
Quicksort Notes

- Each round places one element into position
  - The partitioning element
- Recursive calls handle each half of array
- What would be:
  - a good partitioning element?
  - a bad partitioning element?

Partitioning

- Choose an arbitrary element
  - Any suggestions?
- Place all smaller values to the left
- Place all larger values to the right

Partitioning



C Code: Partitioning

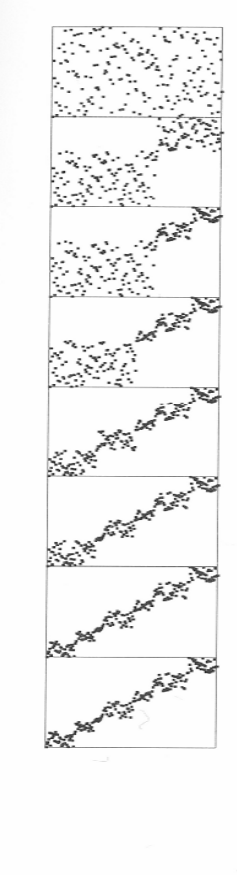
```
int partition(Item a[], int start, int stop)
   {
   int up = start, down = stop – 1, part = a[stop];

   if (stop <= start) return start;

   while (true)
        {
        while (isLess(a[up], part))
            up++;
        while (isLess(part, a[down]) && (up < down))
           down--;

        if (up >= down) break;
        Exchange(a[up], a[down]);
        up++; down--;
        }

   Exchange(a[up], a[stop]);
   return up;
   }
```

Partitioning Notes

- The check (up < down) required when partitioning element is also smallest element.

- N comparisons
  - N - 1 for each element vs. partitioning element
  - One extra is possible when pointers cross

Quick Sort



Array is successively subdivided,
around partitioning element.

Within each section,
items are arranged randomly

Complexity of Quick Sort

- Best case:

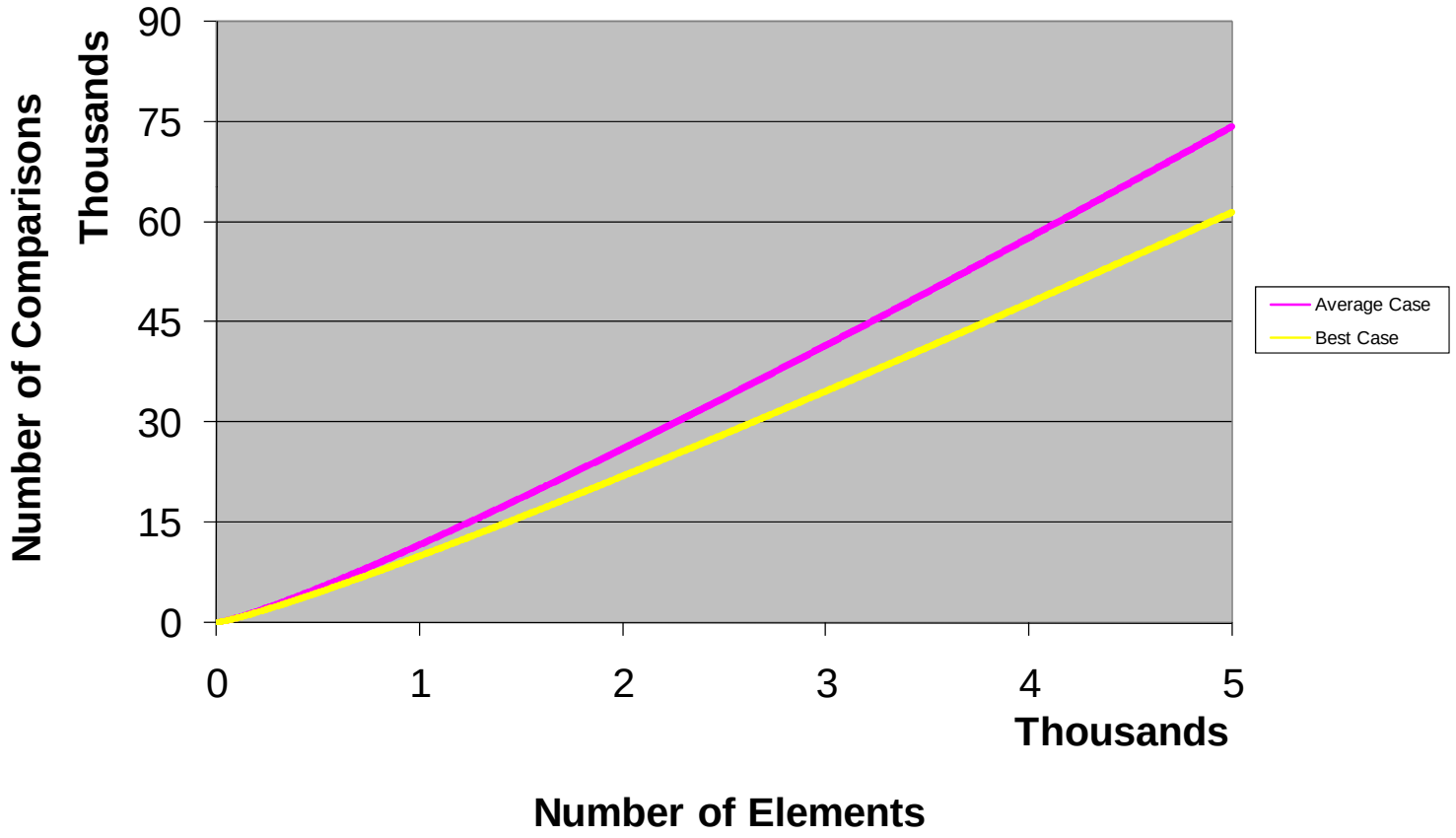
$$C_N = 2C_{N/2} + N = N \log_2 N$$

- Random input:

$$C_N = N + \frac{1}{N} \sum_{k=1}^{N} \{C_{k-1} + C_{N-k}\}$$

$$\approx 2N \log_e N \approx 1.4N \log_2 N$$

Complexity of Quick Sort



Improvements to Quicksort

- Sorting small sub-arrays
  - Quicksort is great for large arrays
  - Inefficient for very small ones
- Choosing a better partitioning element
  - A poor choice could make sort quadratic!

Small Sub-arrays

- Most recursive calls are for small sub-arrays
  - Commonplace for many recursive programs
- “Brute-force” algorithms are often better for small problems
  - In this case, insertion sort is a good option

Sorting Small Sub-arrays

- Possibility 1:
  - ```
    if (stop - start <= M)
      {
      insertion_sort(a, start, stop);
      return;
      }
    ```
- Possibility 2:
  - ```
    if (stop - start <= M) return;
    ```
  - Make a single call to `insertion_sort()` at the end
- 5 < M < 25 gives ~10% speed increase

Sedgewick's Timings

N	Basic	Insertion	Insertion After	Ignore Duplicates	System
12,500	8	7	6	7	10
25,000	16	14	13	17	20
50,000	37	31	31	41	45
100,000	91	78	76	113	103

Arrays including first *N* words in text of "Moby Dick".

Improved Partitioning

- How do we avoid picking smallest or largest element for partitioning?
  - Could take a random element…
  - Could take a sample …

Median-of-Three Partitioning

- Take sample of three elements
- Usually, first, last and middle element
  - Sort these three elements
- Partition around median
  - Very unlikely that worst case would occur

C Code:
Median-of-Three Partitioning

```
void quicksort(Item a[], int start, int stop)
   {
   int i;

   // Leave small subsets for insertion sort
   if (stop - start <= M) return;

   // Place median of 3 in position stop - 1
   Exchange(a[(start + stop)/2], a[stop – 1]);
   CompExch(a[start], a[stop – 1]);
   CompExch(a[start], a[stop]);
   CompExch(a[stop - 1], a[stop]);

   // The first and the last elements are “prepartioned”
   i = partition(a, start + 1, stop - 1);
   quicksort(a, start, i - 1);
   quicksort(a, i + 1, stop);
   }
```

So far...

- Basic Quick Sort
- Median of Three Partitioning
- Brute Force Sorts for Small Problems
- Combined, median-of-three partitioning and insertion sorts for smaller sub-arrays improve performance about 20%

Another Problem ...

- The computer stack has a limited size
- Quick Sort can call itself up to N-1 times
  - Although unusual, deep recursion is possible!
- Can we provide a guarantee on depth of recursion?

The Solution

- Keep track of sections to be solved in "explicit" stack
- After partitioning, handle smaller half first
  - At most, $log_2 N$ smaller halves!

Non-Recursive QuickSort

```
void quicksort(Item a[], int start, int stop)
   {
   int i, s = 0, stack[64];

   stack[s++] = start;
   stack[s++] = stop;

   while (s > 0)
        {
        stop = stack[--s];
        start = stack[--s];
        if (start >= stop) continue;

        i = partition(a, start, stop);
        if (i – start > stop – i)
             {
             stack[s++] = start; stack[s++] = i - 1;
             stack[s++] = i + 1; stack[s++] = stop;
             }
        else {
             stack[s++] = i + 1; stack[s++] = stop;
             stack[s++] = start; stack[s++] = i - 1;
             }
        }
   }
```

Explicit Stacks

- A common feature in computer programs
  - Similar to a “TO DO” list or an “INBOX”
- A simple way to avoid recursion
  - More effort for the programmer
- Another application is in graph traversal

Quick Sort Summary

- Divide and Conquer Algorithm
  - Recursive calls can be “hidden”
- Optimizations
  - Choice of median
  - Threshold for brute-force methods
  - Limiting depth of recursion

A Related Problem

- Consider the problem of finding the k^{th} smallest element in an array

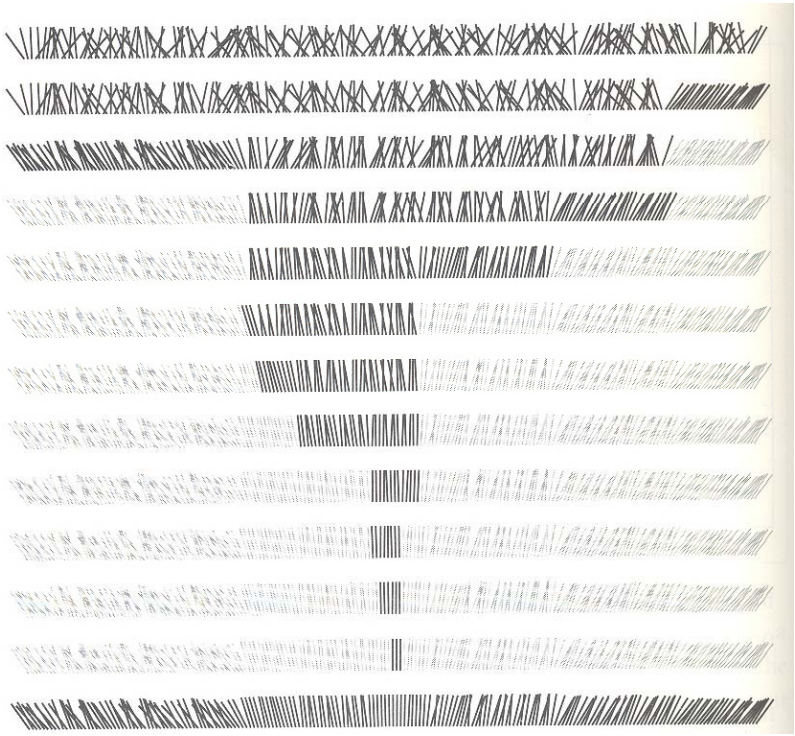
- Useful when searching for the median, quartiles, deciles or percentiles

Selection – small *k*

- We can solve the problem in *O(Nk)* = *O(N)*
- One approach:
  - Perform *k* passes
  - In pass *j*, find *j* smallest element
- Another approach:
  - Maintain a small array with *k* smallest elements

Selection – for large *k*

- One option is to sort array…
- But we only need to bring *k* into position
- Focus on one side of current partition



C Code: Selection

```
// Places kth smallest element in the kth position
// within array. Could move other elements.
void select(Item a[], int start, int stop, int k)
  {
  int i;

  if (stop <= start) return;

  i = partition(a, start, stop);

  if (i > k) select(a, start, i – 1, k);
  if (i < k) select(a, i + 1, stop, k);
  }
```

C Code: Without Recursion

```
// Places kth smallest element in the kth position
// within array. Could move other elements.
void select(Item a[], int start, int stop, int k)
  {
  int i;

  while (start < stop)
      {
      i = partition(a, start, stop);

      if (i >= k) stop = i - 1;
      if (i <= k) start = i + 1;
      }
  }
```

Selection

- Quicksort based method is O(N)
  - Rough argument:
    - First pass through N elements
    - Second pass through N/2 elements
    - Third pass through N/4 elements
    - …
- Common application: finding *k* smallest values in a simulation to save for further analyses

Recommended Reading

- Sedgewick, Chapter 7
- The original description by Hoare (1962) in *Computer Journal* **5:**10-15.