Hashing

Biostatistics 615 / 815

Lecture 9

Scheduling

- Lectures on Hashing:
  - October 6 & 8, 2008
- Review session:
  - October 14, 2008
- Mid-term (in class):
  - October 16, 2008

Last Lecture

- Merge Sort
  - Bottom-Up
  - Top-Down

- Divide and conquer sort with guaranteed *N log N* running time
  - Requires additional auxiliary storage

Today

- Hashing Algorithms
- Fast way to organize data prior to searching
- Trade savings in computing time for additional memory use

Almost Trivia

- Short detour… Finding primes
- How do we find all prime numbers less than some number?

Eratosthenes Sieve

- List all numbers less than N
  - Ignore 0 and 1
- Find the smallest number in the list
  - Mark this number as prime
  - Remove all its multiples from the list
- Repeat previous step until list is empty

The Sieve in C

```
void list_primes()
  {
  int i, j, a[N];

  for (i = 2; i < N; i++)
      a[i] = 1;

  for (i = 2; i < N; i++)
      if (a[i])
         for (j = i * i; j < N; j += i)
            a[j] = 0;

  for (i = 2; i < N; i++)
      if (a[i]) printf("%4d is prime\n", i);
  }
```

Notes on Prime Finding

- The algorithm is extremely fast
  - Takes <1 sec to find all primes <1,000,000
- Performance can be improved by tweaking the inner loop
  - Can you suggest a way?
- Illustrates useful idea:
  - Use values an indices into an array where items denote presence / absence of the value in a set.

Idea

- If all items are integers within a short range…
  - … speed up search operations
  - … avoid having to sort data
- How?

Even better!

- With this strategy…
  - Adding an item to the collection takes constant time
  - Searching through the collection takes constant time
  - Independent of the number of objects in the collection!

Previous Search Strategies

- Place data into an array
  - *O(N)*
- Sort array containing data
  - *O(N log N)*
- Search for items of interest
  - *log N* per search

Using Items as Array Indexes

- Place data into an array
  - *O(N)*
- ~~Sort array containing data~~
- Search for items of interest
  - *O(1)* per search

Another Example ...

- Consider a sorted array with N elements
  - Assume elements uniformly distributed between 0 and 1

- Using binary search, checking for a specific element is about $O(\mathit{log}_2 N)$

- Can we do better by taking into account data is uniformly distributed?

Improving on Binary Search…

- Let N = 100

- What are likely locations for the values
  - 0.0001
  - 0.4281
  - 0.9941

- In fact, we can improve on binary search so that we need only about $\frac{1}{2}\log_2 N$ comparisons.

General Principle

- If the elements we are searching for provide information about their location in the array, we can reduce search times

- We will describe a way to convert an arbitrary element of interest into a likely array location
  - In fact, a series of locations!

Hashing

- Method for converting arbitrary items into array indexes
  - Items can always serve as array indexes…
- A different approach to searching
  - Not (primarily) based on comparisons between elements

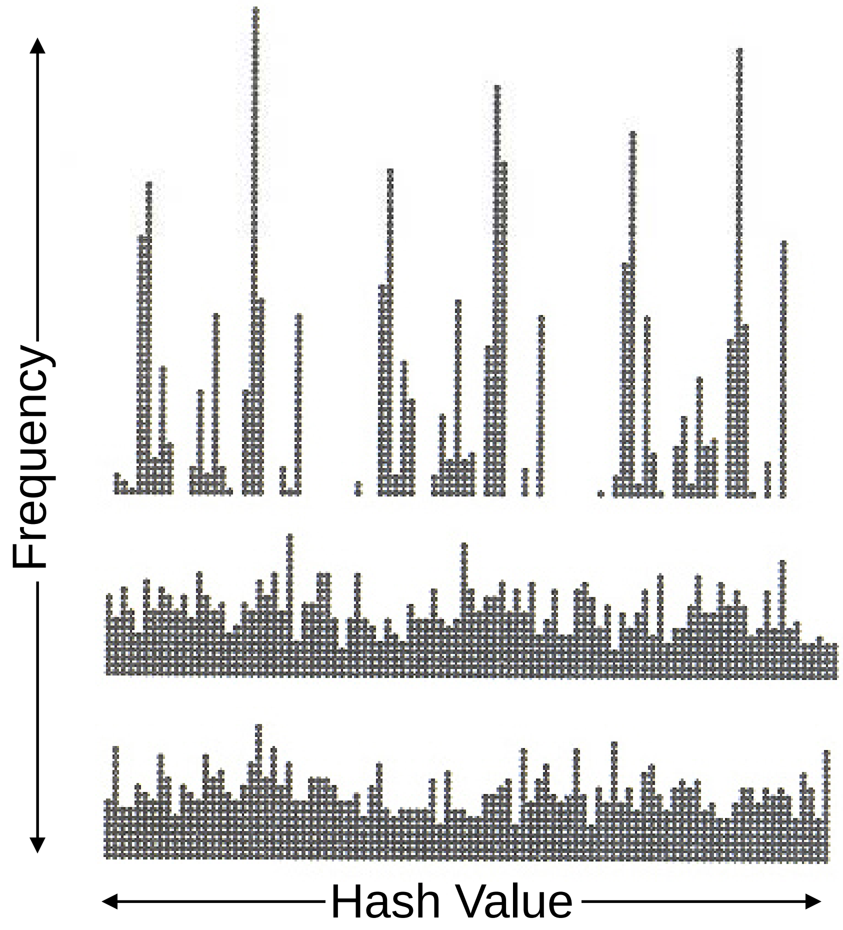
Time – Space Trade Off

- If memory were no issue…
  - Could allocate arbitrarily large array so that each possible item could be a unique index
- If computing time were no issue…
  - Could use linear search to identify matches
- Hashing balances these two extremes

Components of Hashing

- Hash Function
  - Generates table address for individual key
- Collision-Resolution Strategy
  - Deals with keys for which the Hash Function generates identical addresses

Desirable Hash Functions



Poor Hash Function:
Data is clustered

Good Hash Functions:
Data is Evenly Distributed

Hash Function #1

- Assume indexes are in range [0, M – 1]
- Items are floating point values between 0 .. 1
- Multiply by M and round

Hash Function #1

- In general, if items take …
  - Minimum value *min*
  - Maximum value *max*

- Define hash function as

$$(item - min) / (max - min) * M$$

Unfortunately …

- If the items are not randomly distributed within their range…
- Hash function will generate a lot of collisions.
- Better strategies exist…

Hash Function #2

- For integers
- Ensure that table size M is prime
- Define hash function as

item modulus M

Note: The modulus operators is % (in C)

Hash Function #2

- For floating point values
- First map item into 0 .. 1 range
  - As before …
- Multiply result by large integer (say 2^K) and truncate
- Calculate modulus M (where M is prime)

Two Simple Hash Functions

```
int hash_int(int item, int M)
  { return item % M; }

int hash_double(double item, int M)
  {
  return (int) ((item-min)/(max-min)*
                  LARGE_NUMBER) % M;
  }
```

Hashing for Strings

- It is also possible to hash strings …

- One way is to convert each string to a number
  - List all possible characters that could occur
  - Assign the value ‘1’ to one of them, ‘2’ to another, …

- Although the conversion sounds cumbersome, it is built into the way C represents strings
  - Each character is also a number …

A Hash Function for Strings

```
int hash_string(char * s, int M)
  {
  int hash = 0, mult = 127, i;

  for (i = 0; s[i] != 0; i++)
     hash += (s[i] + hash*mult) % M;

  return hash;
  }
```

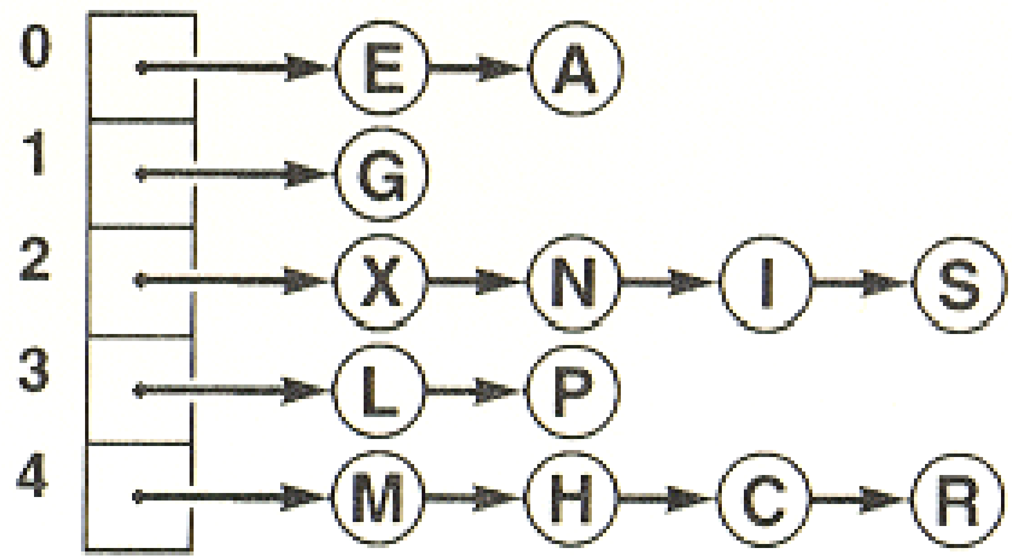
In C, characters can be treated as numbers. A string is an array of characters that terminates with the element zero.

Conflict Resolution: Separate Chaining

- What to do with items where the hash function returns the same value?
- One option is to make each entry in the hash table an array or list …
  - Each entry corresponds to a "chain of items"

Separate Chaining Example

A	S	E	R	C	H	I	N	G	X	M	P	L	Key
0	2	0	4	4	4	2	2	1	2	4	3	3	Hash Value



Chains

Chaining in R
(Relatively Simple Code)

```
# Create a hash table as a list of vectors
table <- vector(M, mode = "list)

for (i in 1:M) table[[i]] <- c()

# Add an element to the table
h <- hash(item, M)
table[[h]] <- c(item, table[[h]])

# Check if an element is in the table
item %in% table[[hash(item, M)]]
```

Chaining in C++ (Certainly More Complicated!)

```
#include "stdio.h"
#include "stdlib.h"

#define EMPTY   -1
#define M        997
#define N        50

class ValueAndPointer
   {
   public:
      int               value;
      ValueAndPointer * next;
   };

ValueAndPointer * hash[M];

void InitializeTable()
   {
   for (int i = 0; i < M; i++)
       hash[i] = NULL;
   }
```

Chaining in C++ (Adding an integer …)

```
void Insert(int value)
   {
   int h = value % M;

   ValueAndPointer ** pointer = &(hash[h]);

   while (*pointer != NULL)
       {
       if ((**pointer).value == value)
          // Value is already in the chain ...
          return;

       pointer = &(**pointer).next;
       }

   // Value not found, add a new link to the chain
   *pointer = (ValueAndPointer *) malloc(sizeof(ValueAndPointer *));

   (**pointer).next = NULL;
   (**pointer).value = value;
   }
```

Chaining in C++ (Finding an integer ...)

```
bool Find(int value)
   {
   int h = value % M;

   ValueAndPointer ** pointer = &(hash[h]);

   while (*pointer != NULL)
       {
       if ((**pointer).value == value)
          return 1;

       pointer = &(**pointer).next;
       }

   return false;
   }
```

Chaining in C++
(Checking the previous functions)

```
int main(int argc, char ** argv)
   {
   srand(123456);

   InitializeTable();

   for (int i = 0; i < N; i++)
      {
      int value = rand() % (N * 10);
      printf("Inserting the value %d into table ...\n", value);
      Insert(value);
      }

   for (int i = 0; i < N; i++)
      {
      int value = rand() % (N * 10);
      printf("Checking for value %d in table ...\n", value);
      if (Find(value))
         printf("    FOUND!!!\n");
      else
         printf("    Not found.\n");
      }
   }
```

Properties of Separate Chaining

- If the hashing function results in random indexes…

$$\frac{N}{M}$$ expected number of entries in each chain

$$\binom{N}{k}\left(\frac{1}{M}\right)^{k}\left(1-\frac{1}{M}\right)^{N-k}$$ number of entries follows Binomial distribution

(well approximated with Poisson distribution)

Interesting Known Properties

- In most slots, no. of entries is close to average

$e^{-\alpha}$	probability of an empty slot
$\sim 1.25\sqrt{M}$	number of items before first collision "the birthday problem"
$\mathrm{M}\left(\sum_{\mathrm{i}=1}^{\mathrm{M}} \frac{1}{\mathrm{i}}\right)$	number of items before all slots have one item "the coupon collector problem"

- α = N / M is the load factor…

Notes on Hashing

- Good performance when
  - Searching for elements
  - Inserting elements

- Ineffective when
  - Selecting elements based on rank
  - Sorting elements

Recommended Reading

- Sedgewick, Chapter 14

- Peterson W. W. (1957) *IBM Journal of Research and Development* **1:**130-146