Biostatistics 615 / 815
Problem Set 7
Due November 17, 2004

Random Number Generation

1. An old text book on computer simulation recommends the following sequence of pseudo-random numbers:

   I_{j+1} **=** a I_j **mod** m, with a = 20,403 and m = 2^{15}.

   What is the period of this random number generator, that is, how many numbers can be generated before the sequence repeats itself?

2. Consider the following formula for generating pseudo-random sequences:

   I_{j+1} **= (**a I_j **+** c**) mod** m

   When c = 1 and m = 32, examine the resulting sequences when:

   a) a = 13
   b) a = 17
   c) a = 19
   d) a = 10

   Which of the three sequences appear random? Do all sequences have a full period (32 is the maximum period in this case)?

Integration

3. Implement code to calculate the integral of the function $f(x) = (1 + x)/(1 + x^2)$ in the interval (2, 4) using each of the following numerical methods:

   a) Standard Trapezoidal rule, with 2 function evaluations.
   b) Trapezoidal rule with 3 function evaluations.
   c) Simpon's rule with 3 function evaluations.
   d) Two point Gaussian quadrature.

   The exact integral is approximately 0.83. How do you explain the differences between the 4 results above?

4. Consider the function *f(x) = sqrt(x) * log(x)* for *x > 0*, and let *f(0) = 0*. Implement code to calculate the integral of this function in the interval (0,1) using each of the following numerical methods:

   a) Adaptive quadrature, using the trapezoidal rule, until a relative precision of 10^{-5} is reached.
   b) Adaptive quadrature, using Simpson's rule, until a relative precision of 10^{-5} is reached.
   c) Four point Gaussian quadrature.

   How many function evaluations where required in each case?