1. Consider the following set of 20 observations drawn from a mixture of two normal distributions.

\[-2.876 \ -0.877 \ 0.728 \ 1.670\]
\[-2.527 \ -0.645 \ 0.737 \ 1.826\]
\[-1.213 \ 0.151 \ 0.819 \ 1.867\]
\[-1.111 \ 0.246 \ 0.998 \ 2.107\]
\[-1.034 \ 0.409 \ 1.602 \ 2.618\]

Assuming that the two distributions have unit variance and symmetric means $\Delta$ and $-\Delta$, the likelihood function for these data is:

$$L(\Delta) = \prod_{i} \frac{1}{\sqrt{2\pi}} \left( e^{-\frac{1}{2}(x_i - \Delta)^2} + e^{-\frac{1}{2}(x_i + \Delta)^2} \right)$$

(The product should be calculated over all observations).

Write a program that:

a) Brackets the maximum of the log-likelihood function.

b) Using the golden-section optimization strategy, finds the MLE for $\Delta$.

c) Using an optimization strategy based on parabolic interpolation, finds the MLE of $\Delta$.

d) How many function evaluations did you need for steps a), b) and c) above?